THE NINE-POINTS CIRCLE.

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ABC is a triangle. A_a is the foot of the altitude from A. A_m is the middle point of the side opposite A. The altitudes intersect at O. A_0 is the middle point of AO.

From similar triangles BCC_a, BAA_a,

$$\begin{aligned} BA_a : BC_a &:: BA : BC \\ &:: BC_m : BA_m; \\ \therefore BA_a . BA_m &= BC_a . BC_m, \end{aligned}$$

and therefore A_a, A_m, C_a, C_m, lie on a circle.

From similar triangles COA_a, CBC_a,

$$\begin{aligned} CO : CA_a &:: CB : CB_a; \\ \therefore CC_0 : CA_a &:: CA_m : CC_a; \\ \therefore CC_0 . CC_a &= CA_a . CA_m; \\ \therefore C_0 &\text{ lies on the circle } A_a, A_m, C_a, \end{aligned}$$

that is, A_a, A_m, C_a, C_m, C_0 lie on a circle;
Similarly, A_a, A_m, C_a, C_m, A_0 " " " "
$\therefore A_a, A_m, C_a, C_m, A_0, C_0$ " " " "
Similarly, $A_a, A_m, B_a, B_m, A_0, B_0$ " " " "
These two circles have three points in common;

$$\therefore A_a, A_m, A_0, B_a, B_m, B_0, C_a, C_m, C_0$$

lie all on the same circle.

SOLUTIONS OF EXERCISES.

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The result

$$-\frac{p^2q^2 + 4p^3r - 8q^3 + 2pqr + 9r^3}{(r - pq)^2}$$

is given as the equivalent of the function

$$\left(\frac{\beta - \gamma}{\beta + \gamma}\right)^2 + \left(\frac{\gamma - \alpha}{\gamma + \alpha}\right)^2 + \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2,$$